Welcome To

Class : :8th

2nd Term syllabus

1. chapter#11:
Trigonometric Ratios

2. chapter#12:
Volume And Surface area of pyramid;Cones and sphere

3. chapter#14:
Sets

Teacher Name:Mrs Zokia

Chapter#11

Trigonometric Ratios

Discussion Topics:

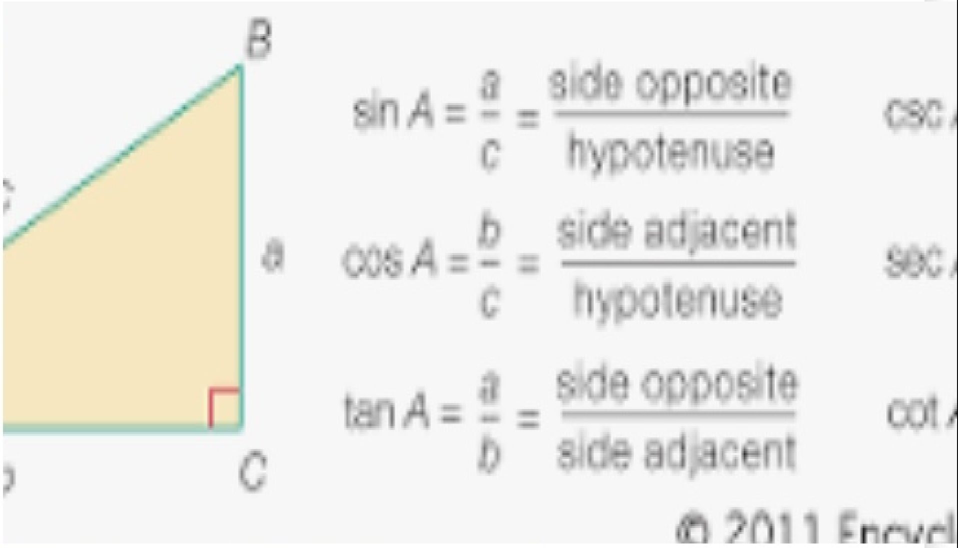
1. Trigonometric Ratios

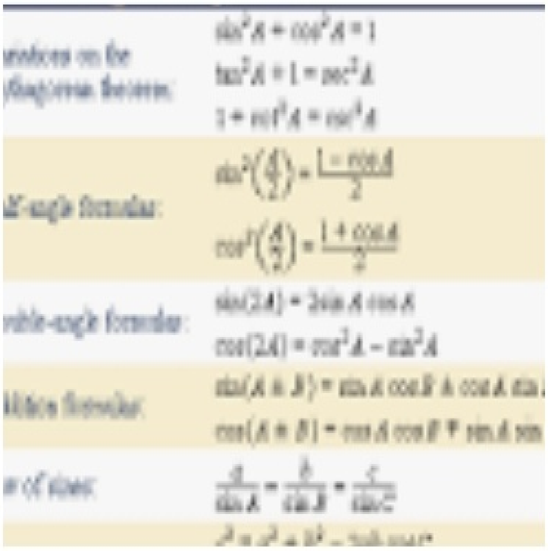
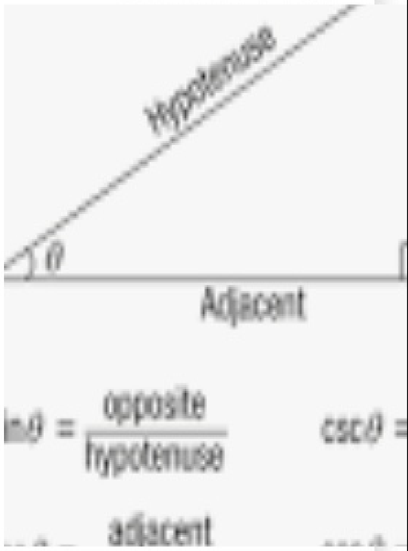
2. How to find unknown sides of Right angled triangles

3. How to find unknown angles in right angled triangles

4. Application of Trigonometric Ratios in real world

Trigonometry definition

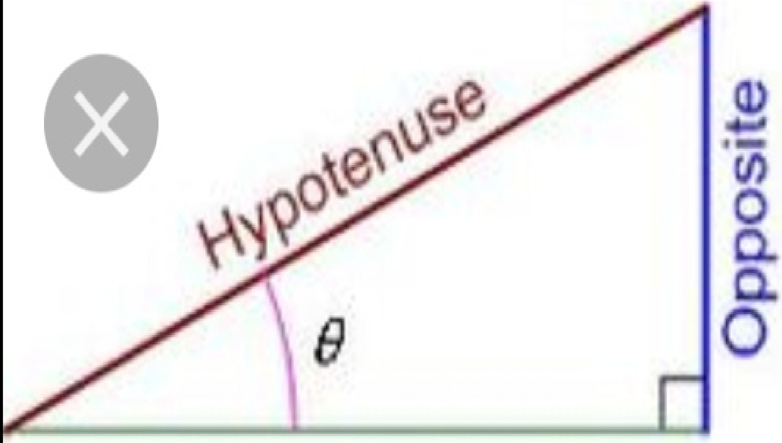


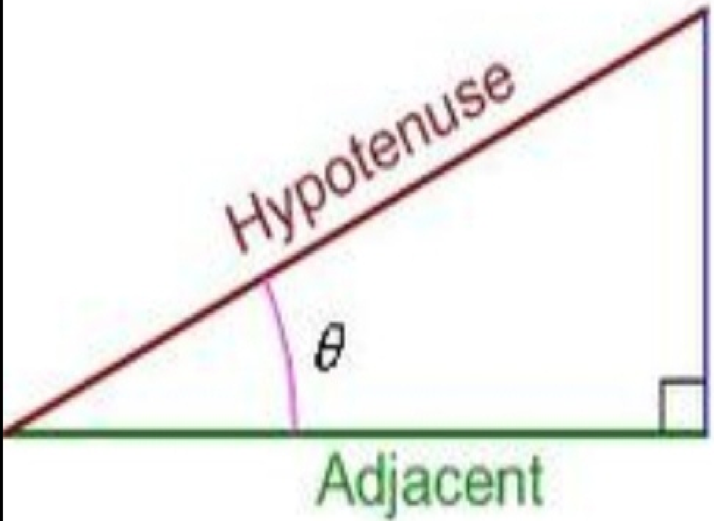
Trigonometry (from Greek trigōnon, "triangle" and metron, "measure") is a branch of mathematics that studies relationships between side lengths and angles of triangles.

Trigonometric Ratios Definition

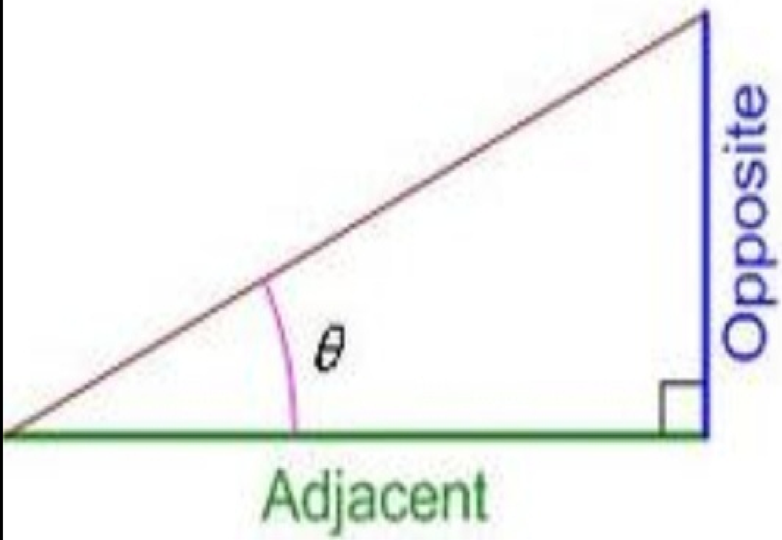
It is defined as the values of all the trigonometric function based on the value of the ratio of sides in a right-angled triangle. **The ratios of sides of a right-angled triangle with respect to any of its acute angles are known as the trigonometric ratios of that particular angle**. Consider a right-angled triangle, right-angled at B.



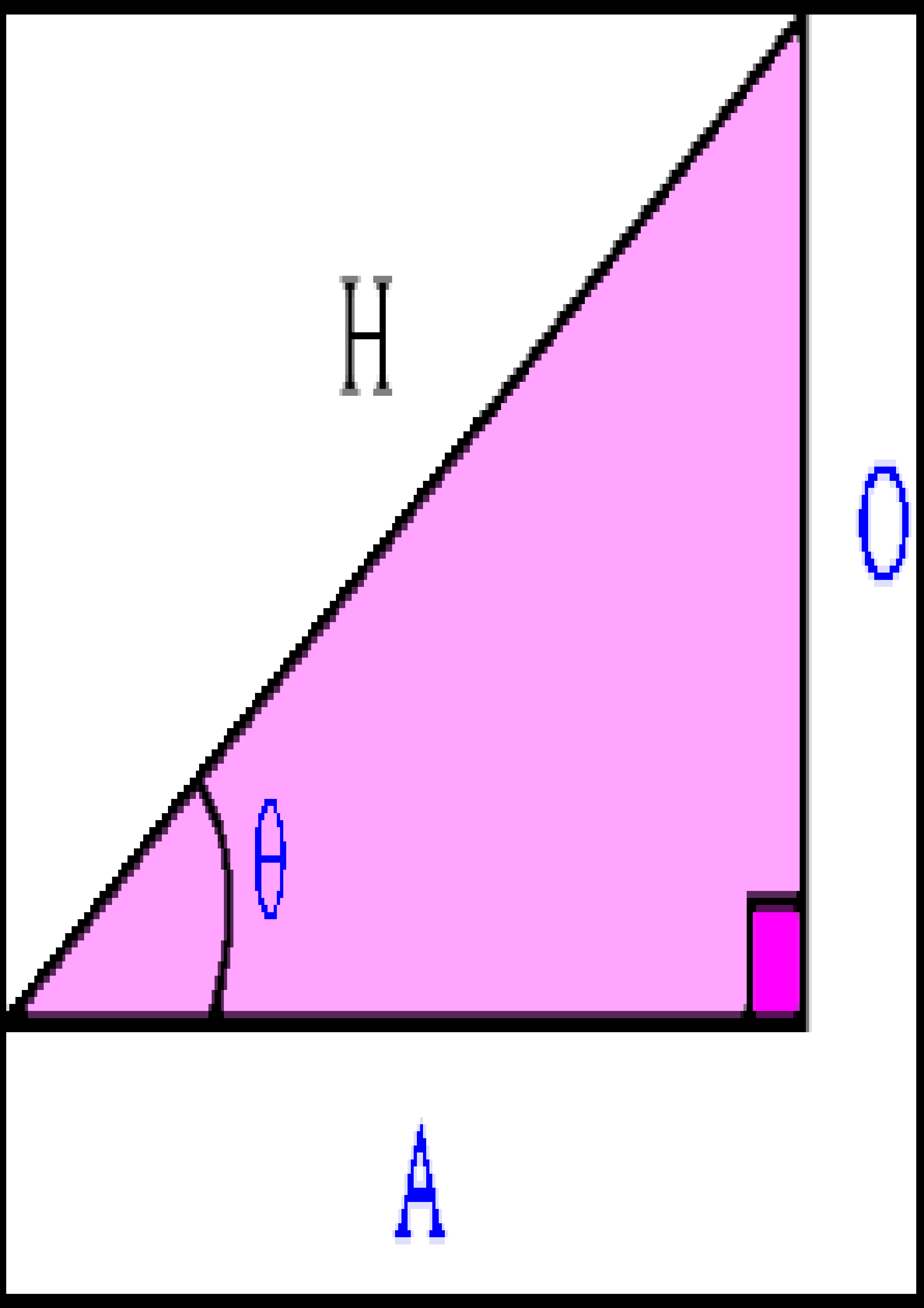
$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$



$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

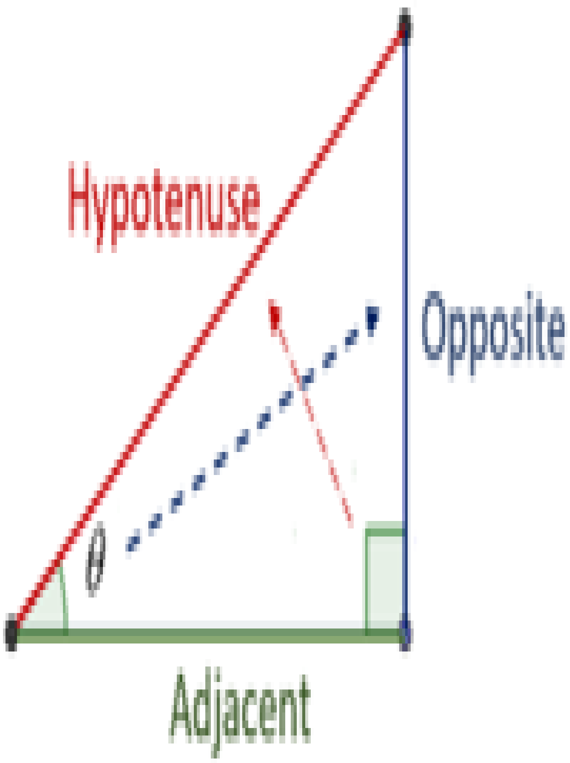


$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$


H
O
θ
A

Trigonometric Ratios

sin, cos, tan, sec, csc, cot



SOH $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

CAH $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

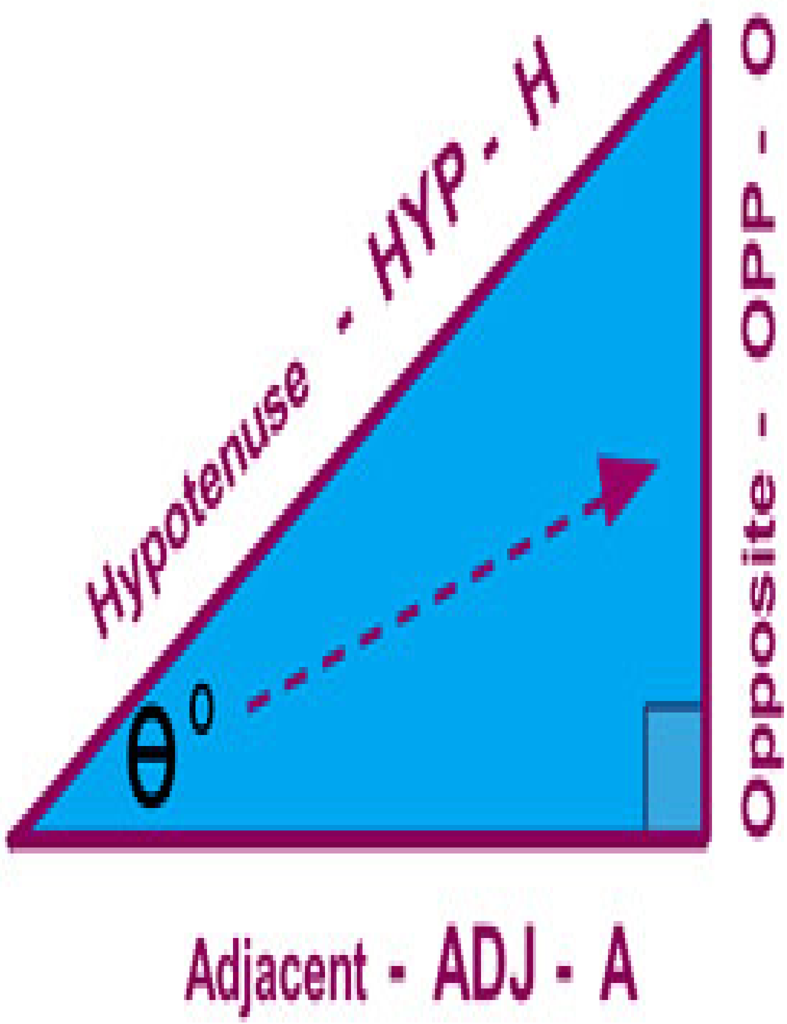
TOA $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

Trig Ratios – SOH CAH TOA



Name	Ratio	Expression
Sine	O / H	SinΘ
Cosine	A / H	CosΘ
Tangent	O / A	TanΘ

<u>We use "SOH-CAH-TOA" to help us remember the Ratios</u>

SOH is short for **Sine** = Opposite / Hypotenuse = O / H

CAH is short for **Cosine** = Adjacent / Hypotenuse = A / H

TOA is short for **Tangent** = Opposite / Adjacent = O / A

Sin Θ (sin Θ)	$\frac{\text{Perpendicular}}{\text{Hypotenuse}}$	$\frac{y}{r}$
Cosine Θ (cos Θ)	$\frac{\text{Base}}{\text{Hypotenuse}}$	$\frac{x}{r}$
Tangent Θ (tan Θ)	$\frac{\text{Perpendicular}}{\text{Base}}$	$\frac{y}{x}$
Cosecant Θ (cossec Θ)	$\frac{\text{Hypotenuse}}{\text{Perpendicular}}$	$\frac{r}{y}$
Secant Θ (sec Θ)	$\frac{\text{Hypotenuse}}{\text{Base}}$	$\frac{r}{x}$
Cotangent Θ (cot Θ)	$\frac{\text{Base}}{\text{Perpendicular}}$	$\frac{x}{y}$

Angles in Degrees	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not defined
csc	Not defined	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

Exercise 11A

1. (a) (i) PQ is the hypotenuse.
   (ii) PR is the side opposite $\angle a$.
   (iii) QR is the side adjacent to $\angle a$.
   (b) (i) XY is the hypotenuse.
   (ii) XZ is the side opposite $\angle a$.
   (iii) YZ is the side adjacent to $\angle a$.

2. (a) (i) $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{5}{13}$

   (ii) $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB} = \frac{12}{13}$

   (iii) $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC} = \frac{5}{12}$

   (iv) $\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{AC}{AB} = \frac{12}{13}$

   (v) $\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{BC}{AB} = \frac{5}{13}$

   (vi) $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{AC}{BC} = \frac{12}{5}$

   (b) (i) $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{24}{25}$

   (ii) $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB} = \frac{7}{25}$

   (iii) $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC} = \frac{24}{7}$

   (iv) $\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{AC}{AB} = \frac{7}{25}$

   (v) $\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{BC}{AB} = \frac{24}{25}$

   (vi) $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{AC}{BC} = \frac{7}{24}$

3. (a) (i) $\sin P = \frac{\text{opp}}{\text{hyp}} = \frac{QR}{PQ} = \frac{y}{z}$

   (ii) $\cos P = \frac{\text{adj}}{\text{hyp}} = \frac{PR}{PQ} = \frac{x}{z}$

   (iii) $\tan P = \frac{\text{opp}}{\text{adj}} = \frac{QR}{PR} = \frac{y}{x}$

   (iv) $\sin Q = \frac{\text{opp}}{\text{hyp}} = \frac{PR}{PQ} = \frac{x}{z}$

(v) $\cos Q = \frac{\text{adj}}{\text{hyp}}$

$= \frac{QR}{PQ}$

$= \frac{y}{z}$

(vi) $\tan Q = \frac{\text{opp}}{\text{adj}}$

$= \frac{PR}{QR}$

$= \frac{x}{y}$

(b) (i) $\sin P = \frac{\text{opp}}{\text{hyp}}$

$= \frac{QR}{PQ}$

$= \frac{z}{x}$

(ii) $\cos P = \frac{\text{adj}}{\text{hyp}}$

$= \frac{PR}{PQ}$

$= \frac{y}{x}$

(iii) $\tan P = \frac{\text{opp}}{\text{adj}}$

$= \frac{QR}{PR}$

$= \frac{z}{y}$

(iv) $\sin Q = \frac{\text{opp}}{\text{hyp}}$

$= \frac{PR}{PQ}$

$= \frac{y}{x}$

(v) $\cos Q = \frac{\text{adj}}{\text{hyp}}$

$= \frac{QR}{PQ}$

$= \frac{z}{x}$

(vi) $\tan Q = \frac{\text{opp}}{\text{adj}}$

$= \frac{PR}{QR}$

$= \frac{y}{z}$

4. (a) Sequence of calculator keys:

[tan] [4] [7] [=]

$\therefore \tan 47° = 1.07$ (to 3 s.f.)

(b) Sequence of calculator keys:

[sin] [7] [5] [.] [3] [=]

$\therefore \sin 75.3° = 0.967$ (to 3 s.f.)

(c) Sequence of calculator keys:

[cos] [3] [0] [.] [1] [9] [=]

$\therefore \cos 30.19° = 0.864$ (to 3 s.f.)

(d) Sequence of calculator keys:

[sin] [3] [5] [+] [cos] [4] [9] [=]

$\therefore \sin 35° + \cos 49° = 1.23$ (to 3 s.f.)

(e) Sequence of calculator keys:

[2] [cos] [4] [2] [.] [3] [+] [3] [sin] [1] [6] [.] [8] [=]

$\therefore 2\cos 42.3° + 3\sin 16.8° = 2.35$ (to 3 s.f.)

(f) Sequence of calculator keys:

[sin] [7] [1] [.] [6] [×] [tan] [1] [6] [.] [7] [=]

$\therefore \sin 71.6° \times \tan 16.7° = 0.285$ (to 3 s.f.)

5. (a) Sequence of calculator keys:

[5] [tan] [6] [1] [.] [4] [÷] [2] [cos] [1] [0] [.] [3] [=]

$\therefore \frac{5\tan 61.4°}{2\cos 10.3°} = 4.66$ (to 3 s.f.)

(b) Sequence of calculator keys:

[4] [(] [sin] [2] [2] [.] [5] [)] [x^2] [÷]
[cos] [6] [7] [.] [5] [=]

$\therefore \frac{4(\sin 22.5°)^2}{\cos 67.5°} = 1.53$ (to 3 s.f.)

(c) Sequence of calculator keys:

[(] [tan] [1] [5] [+] [cos] [3] [3] [)] [÷]
[sin] [7] [8] [.] [4] [=]

$\therefore \frac{\tan 15° + \cos 33°}{\sin 78.4°} = 1.13$ (to 3 s.f.)

(d) Sequence of calculator keys:

[tan] [4] [7] [.] [9] [÷] [(] [cos] [8] [4]
[–] [sin] [6] [3] [)] [=]

$\therefore \frac{\tan 47.9°}{\cos 84° - \sin 63°} = -1.41$ (to 3 s.f.)

(e) Sequence of calculator keys:

[(] [cos] [6] [7] [+] [sin] [8] [9] [)] [÷] [(] [tan]
[6] [3] [.] [4] [×] [cos] [1] [5] [.] [5] [)] [=]

$\therefore \frac{\cos 67° + \sin 89°}{\tan 63.4° \times \cos 15.5°} = 0.723$ (to 3 s.f.)

(f) Sequence of calculator keys:

[(] [sin] [2] [4] [.] [6] [÷] [cos] [6] [2] [.] [1]
[)] [÷] [(] [tan] [2] [1] [+] [cos] [1] [4] [)] [=]

$\therefore \frac{\sin 24.6° \div \cos 62.1°}{\tan 21° + \cos 14°} = 0.657$ (to 3 s.f.)

(g) Sequence of calculator keys:

(sin 5 7 − cos 7 3) ÷ (tan

1 5 . 3 × sin 8 3 . 4) =

$\therefore \dfrac{\sin 57° - \cos 73°}{\tan 15.3° \times \sin 83.4°} = 2.01$ (to 3 s.f.)

(h) Sequence of calculator keys:

(cos 2 4 . 7 × sin 3 5 . 1

) ÷ tan 5 7 − cos 1 5) =

$\therefore \dfrac{\cos 24.7° \times \sin 35.1°}{\tan 57° - \cos 15°} = 0.910$ (to 3 s.f.)

Exercise 11B

1. (a) $\sin 67° = \dfrac{a}{15}$

$\therefore a = 15 \sin 67°$

$= 13.8$ (to 3 s.f.)

(b) $\sin 15° = \dfrac{9.7}{b}$

$\therefore b = \dfrac{9.7}{\sin 15°}$

$= 37.5$ (to 3 s.f.)

2. (a) $\cos 36° = \dfrac{a}{13.5}$

$\therefore a = 13.5 \cos 36°$

$= 10.9$ (to 3 s.f.)

(b) $\cos 61° = \dfrac{17}{b}$

$\therefore b = \dfrac{17}{\cos 61°}$

$= 35.1$ (to 3 s.f.)

3. (a) $\tan 28° = \dfrac{a}{14}$

$\therefore a = 14 \tan 28°$

$= 7.44$ (to 3 s.f.)

(b) $\tan 62.5° = \dfrac{13}{b}$

$\therefore b = \dfrac{13}{\tan 62.5°}$

$= 6.77$ (to 3 s.f.)

4. (a) $\sin 34° = \dfrac{a}{12}$

$\therefore a = 12 \sin 34°$

$= 6.71$ (to 3 s.f.)

$\therefore b = 12 \cos 34°$

$= 9.95$ (to 3 s.f.)

(b) $\cos 43° = \dfrac{c}{16}$

$\therefore c = 16 \cos 43°$

$= 11.7$ (to 3 s.f.)

$\sin 43° = \dfrac{d}{16}$

$\therefore d = 16 \sin 43°$

$= 10.9$ (to 3 s.f.)

(c) $\tan 44.2° = \dfrac{e}{7}$

$\therefore e = 7 \tan 44.2°$

$= 6.81$ (to 3 s.f.)

$\cos 44.2° = \dfrac{7}{f}$

$\therefore f = \dfrac{7}{\cos 44.2°}$

$= 9.76$ (to 3 s.f.)

(d) $\tan 21.5° = \dfrac{8.9}{g}$

$\therefore g = \dfrac{8.9}{\tan 21.5°}$

$= 22.6$ (to 3 s.f.)

$\sin 21.5° = \dfrac{8.9}{h}$

$\therefore h = \dfrac{8.9}{\sin 21.5°}$

$= 24.3$ (to 3 s.f.)

5. (i) In $\triangle ABH$,

$\sin 56° = \dfrac{AH}{8.9}$

$\therefore AH = 8.9 \sin 56°$

$= 7.38$ m (to 3 s.f.)

(ii) Method 1:

In $\triangle ABH$,

$\cos 56° = \dfrac{BH}{8.9}$

$\therefore BH = 8.9 \cos 56°$

$= 4.977$ m (to 4 s.f.)

In $\triangle ABC$,

$\cos 56° = \dfrac{8.9}{BC}$

$\therefore BC = \dfrac{8.9}{\cos 56°}$

$= 15.92$ m (to 4 s.f.)

$\therefore HC = BC - BH$

$= 15.92 - 4.977$

$= 10.9$ m (to 3 s.f.)

Method 2:

In $\triangle ABC$,

$\angle ACB = 180° - 90° - 56°$ ($\angle$ sum of $\triangle ABC$)

$= 34°$

In $\triangle AHC$, $\angle ACH = 34°$.

From **(i)**,

$\tan 34° = \dfrac{7.378}{HC}$

$\therefore HC = \dfrac{7.378}{\tan 34°}$

$= 10.9$ m (to 3 s.f.)

6. (i) In $\triangle QST$, $\angle T = 180° - 90°$ (adj. $\angle$s a str. line)

$= 90°$

$\sin 60° = \frac{TQ}{25}$

$\therefore TQ = 25 \sin 60°$

$= 21.7$ cm (to 3 s.f.)

(ii) **Method 1:**

In $\triangle QST$,

$\cos 60° = \frac{TS}{25}$

$TS = 25 \cos 60°$

$= 12.5$ cm

In $\triangle PQT$, $\angle Q = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

$\cos 60° = \frac{25}{PS}$

$\therefore PS = \frac{25}{\cos 60°}$

$= 50$ cm

$\therefore PT = PS - TS$

$= 50 - 12.5$

$= 37.5$ cm

Method 2:

In $\triangle PQS$,

$\angle PQS = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

$\angle QPS = 180° - 90° - 60°$ ($\angle$ sum of $\triangle PQS$)

$= 30°$

In $\triangle PQT$, $\angle QPT = 30°$.

From (i),

$\tan 30° = \frac{21.65}{PT}$

$\therefore PT = \frac{21.65}{\tan 30°}$

$= 37.5$ cm (to 3 s.f.)

(iii) In $\triangle PQR$,

$\tan 60° = \frac{PQ}{25}$

$\therefore PQ = 25 \tan 60°$

$= 43.30$ cm (to 4 s.f.)

In $\triangle QRS$,

$\tan 45° = \frac{QR}{25}$

$\therefore QR = 25 \tan 45°$

$= 25$ cm

$\therefore PR = PQ + QR$

$= 43.30 + 25$

$= 68.3$ cm (to 3 s.f.)

7. (i) In $\triangle VWX$,

$\cos 63° = \frac{WX}{154}$

$\therefore WX = 154 \cos 63°$

$= 69.91$ m

$\sin 63° = \frac{VX}{154}$

$\therefore VX = 154 \sin 63°$

$= 137.22$ cm

In $\triangle VXY$, $\angle Y = 180° - 90°$ (adj. $\angle$s on a str. line)

$= 90°$

Using Pythagoras' Theorem,

$VX^2 = XY^2 + VY^2$

$137.22^2 = XY^2 + 88^2$

$XY^2 = 137.22^2 - 88^2$

$\therefore XY = \sqrt{137.22^2 - 88^2}$ (since $XY > 0$)

$= 105.29$

In $\triangle VYZ$,

$\tan 46° = \frac{88}{YZ}$

$\therefore YZ = \frac{88}{\tan 46°}$

$= 84.98$

$\sin 46° = \frac{88}{VZ}$

$\therefore VZ = \frac{88}{\sin 46°}$

$= 122.33$

$\therefore$ Perimeter of the figure

$= WX + XY + YZ + VZ + WV$

$= 69.91 + 105.29 + 84.98 + 122.33 + 154$

$= 537$ cm (to 3 s.f.)

(ii) Area of the figure

$=$ Area of $\triangle WXV$ + Area of $\triangle VXZ$

$= \frac{1}{2} \times WX \times VX + \frac{1}{2} \times XZ \times VY$

$= \frac{1}{2} \times 69.91 \times 137.22 + \frac{1}{2} \times (105.29 + 84.98) \times 88$

$= 13\ 200$ m^2 (to 3 s.f.)

8. Since y is inversely proportiona to $(\tan x)^2$,

then $y = \frac{k}{(\tan x)^2}$, where k is a constant.

When $x = 30°$, $y = 2$,

$2 = \frac{k}{(\tan 30°)^2}$

$k = 2(\tan 30°)^2$

$= \frac{2}{3}$

When $x = 2(30°) = 60°$,

$\therefore y = \frac{2}{3} \times \frac{1}{(\tan 60°)^2}$

$= \frac{2}{3} \times \frac{1}{3}$

$= \frac{2}{9}$

Exercise 11C

1. (a) Sequence of calculator keys:

2nd F | sin | 0 | . | 5 | 2 | 7 | =

$\therefore \angle A = \sin^{-1}(0.527)$

$= 31.8°$ (to 1 d.p.)

(b) Sequence of calculator keys:

2nd F | cos | 0 | . | 7 | 2 | 5 | =

$\therefore \angle B = \cos^{-1}(0.725)$

$= 43.5°$ (to 1 d.p.)

(c) Sequence of calculator keys:

2nd F | tan | 2 | . | 5 | 6 | =

$\therefore \angle C = \tan^{-1}(2.56)$

$= 68.7°$ (to 1 d.p.)

2. (a) $\sin a° = \frac{12}{26}$

$\therefore a° = \sin^{-1}\left(\frac{12}{26}\right)$

$= 27.5°$ (to 1 d.p.)

$\therefore a = 27.5$

(b) $\cos b° = \frac{10}{17}$

$\therefore b° = \cos^{-1}\left(\frac{10}{17}\right)$

$= 54.0°$ (to 1 d.p.)

$\therefore b = 54.0$

(c) $\tan c° = \frac{27}{11}$

$\therefore c° = \tan^{-1}\left(\frac{27}{11}\right)$

$= 67.8°$ (to 1 d..)

$\therefore c = 67.8$

(d) $\cos d° = \frac{17.6}{20}$

$\therefore d° = \cos^{-1}\left(\frac{17.6}{20}\right)$

$= 28.4°$ (to 1 d.p.)

$\therefore b = 28.4$

(e) $\sin e° = \frac{15}{22.7}$

$\therefore e° = \sin^{-1}\left(\frac{15}{22.7}\right)$

$= 41.4°$ (to 1 d.p.)

$\therefore e = 41.4$

(f) $\tan f° = \frac{12.5}{14}$

$\therefore f° = \tan^{-1}\left(\frac{12.5}{14}\right)$

$= 41.8°$ (to 1 d.p.)

$\therefore f = 41.8$

(g) $\tan g° = \frac{14.7}{12.9}$

$\therefore g° = \tan^{-1}\left(\frac{14.7}{12.9}\right)$

$= 48.7°$ (to 1 d.p.)

$\therefore g = 48.7$

(h) $\cos h° = \frac{15.8}{21.2}$

$\therefore h° = \cos^{-1}\left(\frac{15.8}{21.2}\right)$

$= 41.8°$ (to 1 d.p.)

$\therefore h = 41.8$

(i) $\sin i° = \frac{32.75}{41.62}$

$\therefore i° = \sin^{-1}\left(\frac{32.75}{41.62}\right)$

$= 51.9°$ (to 1 d.p.)

$\therefore i = 51.9$

3. (i) Let the point where the perpendicular line from A to CD meets CD be X.

In $\triangle AXD$,

$\angle AXD = 90°$ and $XD = (7 - 4) = 3$ m

$\cos \angle ADC = \frac{3}{7}$

$\therefore \angle ADC = \cos^{-1}\left(\frac{3}{7}\right)$

$= 64.6°$ (to 1 d.p.)

(ii) In $\triangle AXD$, $\angle X = 90°$.

Using Pythagoras' Theorem,

$AD^2 = AX^2 + XD^2$

$7^2 = AX^2 + 3^2$

$AX^2 = 7^2 - 3^2$

$= 49 - 9$

$= 40$

$\therefore AX = \sqrt{40}$ (since $AX > 0$)

$= 6.32$ m (to 3 s.f.)

$\therefore BC = AX$

$= 6.32$ m

4. (i) In $\triangle HMN$,

$\cos 38° = \frac{MN}{9.2}$

$\therefore MN = 9.2 \cos 38°$

$= 7.250$ cm (to 4 s.f.)

In $\triangle LMN$,

$\sin \angle MLN = \frac{7.250}{15.5}$

$\therefore \angle MLN = \sin^{-1}\left(\frac{7.250}{15.5}\right)$

$= 27.9°$ (to 1 d.p.)

(ii) In $\triangle HMN$,

$\sin 38° = \frac{HN}{9.2}$

$\therefore HN = 9.2 \sin 38°$

$= 5.664$ cm (to 4 s.f.)

In $\triangle LMN$, $\angle N = 90°$.

Using Pythagoras' Theorem,

$LM^2 = MN^2 + LN^2$

$15.5^2 = 7.250^2 + LN^2$

$LN^2 = 15.5^2 - 7.250^2$

$= 240.25 - 52.5625$

$= 187.6875$

$\therefore LN = \sqrt{187.6875}$ (since $LN > 0$)

$= 13.70$ cm (to 4 s.f.)

$\therefore HL = LN - HN$

$= 13.70 - 5.664$

$= 8.04$ cm (to 3 s.f.)

5. (i) In $\triangle PQR$,

$\sin \angle PQR = \frac{7.6}{17.4}$

$\therefore \angle PQR = \sin^{-1}\left(\frac{7.6}{17.4}\right)$

$= 25.90°$ (to 2 d.p.)

In $\triangle PQK$,

$\angle QPK = 180° - 137° - 25.90°$ ($\angle$ sum of $\triangle PQK$)

$= 17.1°$ (to 1 d.p.)

(ii) In $\triangle PKR$,

$\angle PRK = 180° - 137°$ (adj. $\angle$s on a str. line)

$= 43°$

$\tan 43° = \frac{7.6}{KR}$

$\therefore KR = \frac{7.6}{\tan 43°}$

$= 8.150$ m (to 4 s.f.)

In $\triangle PQR$, $\angle R = 90°$.

Using Pythagoras' Theorem,

$PQ^2 = QR^2 + PR^2$

$17.4^2 = QR^2 + 7.6^2$

$QR^2 = 17.4^2 - 7.6^2$

$= 302.76 - 57.76$

$= 245$

$\therefore QR = \sqrt{245}$ (since $QR > 0$)

$= 15.65$ m (to 4 s.f.)

$\therefore QK = QR - KR$

$= 15.65 - 8.150$

$= 7.50$ m (to 3 s.f.)

6. $\frac{TH}{HU} \times 100\% = 120\%$

$\frac{TH}{HU} = \frac{6}{5}$

$\therefore TH = \frac{6}{5+6} \times TU$

$= \frac{6}{11} \times 11$

$= 6$ cm

$HU = 11 - 6$

$= 5$ cm

In $\triangle STH$,

Area of $\triangle STH = 21$

$\frac{1}{2} \times 6 \times SH = 21$

$SH = 7$

$\tan \angle STH = \frac{7}{6}$

$\therefore \angle STH = \tan^{-1}\left(\frac{7}{6}\right)$

$= 49.40°$ (to 2 d.p.)

In $\triangle SUH$,

$\tan \angle SUH = \frac{7}{5}$

$\therefore \angle SUH = \tan^{-1}\left(\frac{7}{5}\right)$

$= 54.46°$ (to 2 d.p.)

In $\triangle STU$,

$\angle TSU = 180° - 49.40° - 54.46°$ ($\angle$ sum of $\triangle STU$)

$= 76.1°$ (to 1 d.p.)

7. (i) $\frac{WK}{ZY} = \frac{6}{13}$

$\therefore WK = \frac{6}{13} \times 7.8$

$= 3.6$ m

In $\triangle KWZ$,

$\tan 62° = \frac{3.6}{KZ}$

$\therefore KZ = \frac{3.6}{\tan 62°}$

$= 1.914$ m (to 4 s.f.)

Let the point where the perpendicular line from X to ZY meets ZY be L.

$LY = 7.8 - 1.914 - 4.7$

$= 1.186$ m

In $\triangle LXY$,

$\tan \angle XYL = \frac{3.6}{1.186}$

$\tan \angle XYZ = \frac{3.6}{1.186}$

$\therefore \angle XYZ = \tan^{-1}\left(\frac{3.6}{1.186}\right)$

$= 71.8°$ (to 1 d.p.)

(ii) In $\triangle KWZ$,

$$\sin 62° = \frac{3.6}{WZ}$$

$$\therefore WZ = \frac{3.6}{\sin 62°}$$

$$= 4.077 \text{ m (to 4 s.f.)}$$

In $\triangle LXY$,

$$\sin 71.77° = \frac{3.6}{XY}$$

$$\therefore XY = \frac{3.6}{\sin 71.77}$$

$$= 3.790 \text{ m (to 4 s.f.)}$$

$\therefore$ Perimeter of the trapezium $WXYZ$

$= WX + XY + YZ + WZ$

$= 4.7 + 3.790 + 7.8 + 4.077$

$= 20.4$ m (to 3 s.f.)

8. Let AH be h units.

In $\triangle ABH$,

$$\tan 35° = \frac{h}{BH}$$

$$\therefore BH = \frac{h}{\tan 35°}$$

$$HC = \frac{2h}{\tan 35°}$$

In $\triangle ACH$,

$$\tan \angle ACH = \frac{h}{\frac{2h}{\tan 35°}}$$

$$\tan \angle ACB = \frac{\tan 35°}{2}$$

$$\therefore \angle ACB = \tan^{-1}\left(\frac{\tan 35°}{2}\right)$$

$$= 19.3° \text{ (to 1 d.p.)}$$

Exercise 11D

1. $\tan 32° = \dfrac{TB}{34}$

$\therefore TB = 34 \tan 32°$

$= 21.2$ m (to 3 s.f.)

The height of the Christmas tree is 21.2 m.

2. $\tan 27° = \dfrac{7.7}{AQ}$

$\therefore AQ = \dfrac{7.7}{\tan 27°}$

$= 15.1$ m (to 3 s.f.)

The distance AQ is 15.1 m.

3. $\cos 53° = \dfrac{AB}{120}$

$\therefore AB = 120 \cos 53°$

$= 72.2$ m (to 3 s.f.)

The distance AB is 72.2 m.

4. $\tan \angle PRQ = \dfrac{82}{62}$

$\therefore \angle PRQ = \tan^{-1}\left(\dfrac{82}{62}\right)$

$= 52.9°$ (to 1 d.p.)

5. (i) Let the height of the nail above the ground be h m.

$\sin 60° = \dfrac{h}{5}$

$\therefore h = 5 \sin 60°$

$= 4.33$ (to 3 s.f.)

The nail is 4.33 m above the ground.

(ii) Let the distance of the foot of the ladder from the base of the wall be d m.

$\cos 60° = \dfrac{d}{5}$

$\therefore d = 5 \cos 60°$

$= 2.5$

The distance of the foot of the ladder from the base of the wall is 2.5 m.

6. Let the angle the rope makes with the water be $a°$.

$\sin a° = \dfrac{3.5}{12}$

$\therefore a° = \sin^{-1}\left(\dfrac{3.5}{12}\right)$

$= 17.0°$ (to 1 d.p.)

The rope makes an angle of 170° with the water.

7. Let the point where the horizontal line from Lixin's eyes meets the status be X.

$\tan 42° = \dfrac{MX}{7.05}$

$\therefore MX = 7.05 \tan 42°$

$= 6.348$ m (to 4 s.f.)

$\therefore$ Height of the statue $= 6.348 + 1.55$

$= 7.90$ m (to 3 s.f.)

The height of the statue is 7.90 m.

8. In $\triangle HRW$,

$\tan 24.3° = \dfrac{8}{HR}$

$\therefore HR = \dfrac{8}{\tan 24.3°}$

$= 17.72$ m (to 4 s.f.)

$WQ = HR = 17.72$ m

In $\triangle PQW$,

$\tan 35.4° = \dfrac{PQ}{17.72}$

$\therefore PQ = 17.72 \tan 35.4°$

$= 12.59$ m (to 4 s.f.)

$\therefore$ Height of the flagpole PR

$= PQ + QR$

$= 12.59 + 8$

$= 20.6$ m (to 3 s.f.)

9. Let the angle the plank makes with the wall be $a°$.

$\cos a° = \dfrac{1.8}{4 - 1.2}$

$\therefore a° = \cos^{-1}\left(\dfrac{1.8}{2.8}\right)$

$= 50.0°$ (to 1 d.p.)

The plank makes an angle of 50.0° with the wall.

10. Let the height in which the pendulum rises above Y be h cm.

$\cos\left(\dfrac{30°}{2}\right) = \dfrac{45 - h}{45}$

$\cos 15° = \dfrac{45 - h}{45}$

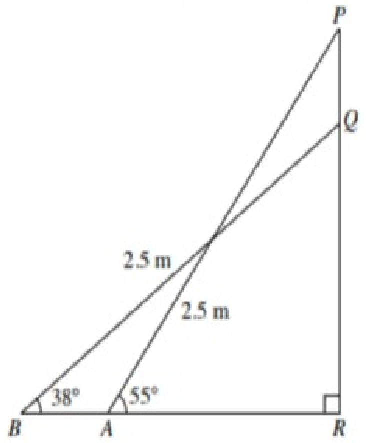
$45 \cos 15° = 45 - h$

$\therefore h = 45 - 45 \cos 15°$

$= 1.53$ m (to 3 s.f.)

The height is 1.53 cm.

11. In the figure, AP and BQ represent the two positions of the ladder and PQ represents the window.



In $\triangle APR$,

$\sin 55° = \dfrac{PR}{2.5}$

$\therefore PR = 2.5 \sin 55°$

$= 2.048$ m (to 4 s.f.)

In $\triangle BQR$,

$\sin 38° = \dfrac{QR}{2.5}$

$\therefore QR = 2.5 \sin 38°$

$= 1.539$ (to 4 s.f.)

$\therefore PQ = PQ - QR$

$= 2.048 - 1.539$

$= 0.509$ m (to 3 s.f.)

$= 50.9$ cm

The height of the window is 50.9 cm.

12. (i) In $\triangle ACH$,

$\sin 35° = \dfrac{CH}{36}$

$\therefore CH = 36 \sin 35°$

$\therefore CD = 36 \sin 35° + 18$

$= 38.6$ m (to 3 s.f.)

(ii) In $\triangle ACH$,

$\cos 35° = \dfrac{AH}{36}$

$\therefore AH = 36 \cos 35°$

$\therefore AF = AH - FH$

$= AH - GD$

$= 36 \cos 35° - 20$

In $\triangle AEF$, $\angle A = 90°$.

Using Pythagoras' Theorem,

$AE^2 = AF^2 + EF^2$

$36^2 = (36 \cos 35° - 20)^2 + EF^2$

$EF^2 = 36^2 - (36 \cos 35° - 20)^2$

$\therefore EF = \sqrt{36^2 - (36\cos 35° - 20)^2}$ (since $EF > 0$)

$= 34.7$ m (to 3 s.f.)

(iii) In $\triangle AEF$,

$\cos \angle EAF = \dfrac{36\cos 35° - 20}{36}$

$\therefore \angle EAF = \cos^{-1}\left(\dfrac{36\cos 35° - 20}{36}\right)$

$= 74.72°$ (to 2 d.p.)

$\therefore$ Angle in which the jib has rotated

$= \angle EAC$

$= 74.72° - 35°$

$= 39.7°$ (to 1 d.p.)

13. Let the height of the tree be h m,

the distance QB be d m.

In $\triangle TBQ$,

$\tan 32° = \dfrac{h}{d}$

$\therefore d = \dfrac{h}{\tan 32°}$ — (1)

In $\triangle TBP$,

$\tan 23° = \dfrac{h}{d + 10}$

$d + 10 = \dfrac{h}{\tan 23°}$

$\therefore d = \dfrac{h}{\tan 23°} - 10$ — (2)

(1) = (2):

$\dfrac{h}{\tan 32°} = \dfrac{h}{\tan 23°} - 10$

$h \tan 23° = h \tan 32° - 10 \tan 32° \tan 23°$

$h \tan 32° - h \tan 23° = 10 \tan 32° \tan 23°$

$h(\tan 32° - \tan 23°) = 10 \tan 32° \tan 23°$

$\therefore h = \dfrac{10 \tan 32° \tan 23°}{\tan 32° - \tan 23°}$

$= 13.2$ (to 3 s.f.)

The height of the tree is 13.2 m.