

Bahria College Zafar Campus E-8 Islamabad

Class: 9th

Unit # 03

Logarithm

MEMORIZE THEM!!!

Exponential Reasoning

$$[1] \quad \log_a 1 = 0$$

$$a^0 = 1$$

$$[2] \quad \log_a a = 1$$

$$a^1 = a$$

$$[3] \quad \log_a a^x = x$$

$$a^x = a^x$$

Logarithmic Properties

Product Rule

$$\log_a(xy) = \log_a x + \log_a y$$

Quotient Rule

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Power Rule

$$\log_a x^p = p \log_a x$$

Change of Base Rule

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Equality Rule

$$\text{If } \log_a x = \log_a y \text{ then } x = y$$

Exercise 3.3:

Q1. Write the following into sum or difference.

(i) $\log(A \times B)$

By 1st law of logarithm:

$$\log(A \times B) = \log A + \log B =$$

(ii) $\log \frac{15.2}{30.5}$

By 2nd law of logarithm:

$$\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5$$

(iii) $\log \frac{21 \times 5}{8}$

By 2nd law of logarithm:

$$\log \frac{21 \times 5}{8} = \log(21 \times 5) - \log 8$$

By 1st law of logarithm:

$$\log \frac{21 \times 5}{8} = \log 21 + \log 5 - \log 8$$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

$$= \log \left(\frac{7}{15} \right)^{1/3}$$

By 3rd law of logarithm:

$$= \frac{1}{3} \log \frac{7}{15}$$

By 2nd law of logarithm:

$$= \frac{1}{3} \log 7 - \frac{1}{3} \log 15$$

(v) $\log \frac{22^{\frac{1}{3}}}{5^3}$

By 2nd law of logarithm:

$$= \log 22^{\frac{1}{3}} - \log 5^3$$

By 3rd law of logarithm:

$$= \frac{1}{3} \log 22 - 3 \log 5$$

(vi) $\log \frac{25 \times 47}{29}$

By 2nd law of logarithm:

$$= \log(25 \times 47) - \log 29$$

By 1st law of logarithm:

$$= \log 25 + \log 47 - \log 29$$

Q3. Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$

$$= \log(21 \times 5)$$

$$= \log 105$$

(ii) $\log 25 + 2\log 3$

$$= \log 25 + \log 3^2$$

$$= \log 25 + \log 9$$

$$= \log(25 \times 9)$$

$$= \log 225$$

(iii) $2\log x - 3\log y$

$$= \log x^2 - \log y^3$$

$$= \log \frac{x^2}{y^3}$$

(iv) $\log 5 + \log 6 - \log 2$

$$= \log \frac{5 \times 6}{2}$$

$$= \log 15$$

Q2. Express $\log x - 2 \log x + 3 \log(x + 1) - \log(x^2 - 1)$ as a single logarithm.

By re arranging

$$\begin{aligned} &= \log x + 3 \log(x + 1) - \log(x^2 - 1) - 2 \log x \\ &= \log x + 3 \log(x + 1) - \{\log(x^2 - 1) + 2 \log x\} \end{aligned}$$

By 3rd law:

$$= \log x + \log(x + 1)^3 - \{\log(x^2 - 1) + \log x^2\}$$

By 1st law:

$$= \log x(x + 1)^3 - \{\log x^2(x^2 - 1)\}$$

By 2nd law:

$$= \log \frac{x(x + 1)^3}{x^2(x^2 - 1)}$$

Simplifying

$$= \log \frac{(x + 1)^3}{x(x - 1)(x + 1)}$$

$$= \log \frac{(x + 1)^2}{x(x - 1)}$$

Q4. Calculate the following:

(i) $\log_3 2 \times \log_2 81$

by 4th law:

$$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

simplifying

$$= \frac{\log 3^4}{\log 3}$$

by 1st law:

$$= \frac{4 \log 3}{\log 3}$$

$$= 4$$

(ii) $\log_5 3 \times \log_3 25$

by 4th law:

$$= \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$$

simplifying

$$= \frac{\log 5^2}{\log 5}$$

by 1st law:

$$= \frac{2 \log 5}{\log 5}$$

$$= 2$$

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$,
 $\log 5 = 0.6990$, then find the
values of the following.

i) $\log 32$ ii) $\log 24$

iii) $\log \sqrt{3\frac{1}{3}}$ iv) $\log \frac{8}{3}$

v) $\log 30$

Solution

i) $\log 32$

$$\log_3 32$$

$$= \log (2 \times 2 \times 2 \times 2 \times 2)$$

$$= 5 \log 2^5$$

$$= 5 \log 2$$

$$= 5 (0.3010) = 1.5050$$

Handwritten notes:
 $\log(2^5)$
 $5 \log(2)$

ii) $\log 24$

$$\log 24$$

$$= \log 8 \times 3$$

$$= \log 8 + \log 3$$

$$= \log 2^3 + \log 3$$

$$= 3 \log 2 + \log 3$$

$$= 3(0.3010) + 0.4771$$

$$= 0.9030 + 0.4771$$

$$= 1.3801$$

iii) $\log \sqrt{3\frac{1}{3}}$

$$= \log \left(\frac{10}{3} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log \frac{2 \times 5}{3}$$

$$= \frac{1}{2} [\log 2 + \log 5 - \log 3]$$

$$= [0.3010 + 0.6990 - 0.4771]$$

$$= \frac{1}{2} [0.5230] = 0.2615$$

iv) $\log \frac{8}{3}$

$$\log \frac{2^3}{3}$$

$$= \log 2^3 \times \log 3$$

$$= 3 \log 2 - \log 3$$

$$= 3 (0.3010) - 0.4771$$

$$= 0.9030 - 0.4771$$

$$= 0.4259$$

v) $\log 30$

$$\log (2 \times 3 \times 5)$$

$$= \log 2 + \log 3 + \log 5$$

$$= 0.3010 + 0.4771 + 0.6990$$

$$= 1.4771$$