Exercise 6C

1. **(a)**

$$ax + by = k$$

$$ax + by - ax = k - ax$$

$$by = k - ax$$

$$\frac{by}{b} = \frac{k - ax}{b}$$

$$\therefore y = \frac{k - ax}{b}$$

(b) $PV = nRT$

$$\frac{PV}{RT} = \frac{nRT}{RT}$$

$$\therefore n = \frac{PV}{RT}$$

Q#1

(c) $5b - 2d = 3c$

$5b - 2d + 2d = 3c + 2d$

$5b = 3c + 2d$

$5b - 3c = 3c + 2d - 3c$

$5b - 3c = 2d$

$\frac{5b - 3c}{2} = \frac{2d}{2}$

$\therefore d = \frac{5b - 3c}{2}$

(d) $R = m(a + g)$

$\frac{R}{m} = \frac{m(a + g)}{m}$

$\frac{R}{m} = a + g$

$\frac{R}{m} - g = a + g - g$

$\therefore a = \frac{R}{m} - g$

2. (a) $\frac{a}{m} = b + c$

$m \times \frac{a}{m} = m \times (b + c)$

$\therefore a = m(b + c)$

(b) $5q - r = \frac{2p}{3}$

$\frac{3}{2} \times (5p - r) = \frac{3}{2} \times \frac{2p}{3}$

$\therefore p = \frac{3}{2}(5p - r)$

(c) $\frac{k + a}{5} = 3k$

$5 \times \frac{k + a}{5} = 5 \times 3k$

$k + a = 15k$

$k + a - k = 15k - k$

$a = 14k$

$\frac{a}{14} = \frac{14k}{14}$

$\therefore k = \frac{a}{14}$

(d) $A = \frac{1}{2}(a + b)h$

$2 \times A = 2 \times \frac{1}{2}(a + b)h$

$2A = (a + b)h$

$\frac{2A}{h} = \frac{(a + b)h}{h}$

$\frac{2A}{h} = a + b$

$\frac{2A}{h} - a = a + b - a$

$\therefore b = \frac{2A}{h} - a$

3. (a) $\sqrt[3]{h - k} = m$

$h - k = m^3$

$h - k + k = m^3 + k$

$\therefore h = m^3 + k$

(b) $b = \sqrt{D + 4ac}$

$b^2 = D + 4ac$

$b^2 - 4ac = D + 4ac - 4ac$

$\therefore D = b^2 - 4ac$

(c) $P = \frac{V^2}{R}$

$P \times R = \frac{V^2}{R} \times R$

$PR = V^2$

$\pm\sqrt{PR} = V$

$\therefore V = \pm\sqrt{PR}$

(d) $A = \frac{\theta}{360} \times \pi r^2$

$360 \times A = 360 \times \frac{\theta}{360} \times \pi r^2$

$360A = \theta \times \pi r^2$

$\frac{360A}{\pi r^2} = \frac{\theta \times \pi r^2}{\pi r^2}$

$\therefore \theta = \frac{360A}{\pi r^2}$

4. When $a = 2$, $b = 7$ and $c = 5$,

$\sqrt{2x^2 - 7} = 5$

$2x^2 - 7 = 25$

$2x^2 = 32$

$x^2 = 16$

$\therefore x = \pm\sqrt{16}$

$= \pm 4$

5. (a) When $b = 7$ and $c = 2$,

$a = \sqrt{\frac{3(7) + 2}{7 - 2}}$

$= \sqrt{\frac{21 + 2}{7 - 2}}$

$= \sqrt{\frac{23}{5}}$

(b) When $a = 4$ and $b = 9$,

$4 = \sqrt{\frac{3(9) + c}{9 - c}}$

$= \sqrt{\frac{27 + c}{9 - c}}$

$16 = \frac{27 + c}{9 - c}$

$16(9 - c) = 27 + c$

$144 - 16c = 27 + c$

$144 = 27 + 17c$

$117 = 17c$

$\therefore c = 6\frac{15}{17}$

6. **(a)** $\frac{a}{a+2} = \frac{3}{5}$

$5a = 3(a+2)$

$5a = 3a + 6$

$2a = 6$

$\therefore a = 3$

(b) $\frac{1}{b-2} = \frac{2}{b-1}$

$b - 1 = 2(b-2)$

$b - 1 = 2b - 4$

$b + 3 = 2b$

$\therefore b = 3$

(c) $\frac{4}{c+3} - \frac{3}{c+2} = 0$

$4(c+2) - 3(c+3) = 0$

$4c + 8 - 3c - 9 = 0$

$c - 1 = 0$

$\therefore c = 1$

(d) $\frac{5}{d+4} - \frac{2}{d-2} = 0$

$5(d-2) - 2(d+4) = 0$

$5d - 10 - 2d - 8 = 0$

$3d - 18 = 0$

$3d = 18$

$\therefore d = 6$

(e) $\frac{6}{f} - \frac{10}{3f} = 2$

$18 - 10 = 6f$

$8 = 6f$

$\therefore f = 1\frac{1}{3}$

(f) $\frac{5}{6h} + \frac{6}{7h} - \frac{9}{14h} = 4$

$35 + 36 - 27 = 168h$

$44 = 168h$

$\therefore h = \frac{11}{42}$

(g) $\frac{3}{k+1} - \frac{1}{2k+2} = 5$

$6 - 1 = 5(2k+2)$

$5 = 10k + 10$

$-5 = 10k$

$\therefore k = -\frac{1}{2}$

7. **(a)** $F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C + 32 - 32$

$F - 32 = \frac{9}{5}C$

$\frac{5}{9} \times (F - 32) = \frac{5}{9} \times \frac{9}{5}C$

$\therefore C = \frac{5}{9}(F - 32)$

(b) $A = 2\pi r^2 + \pi rl$

$A - 2\pi r^2 = 2\pi r^2 + \pi rl - 2\pi r^2$

$A - 2\pi r^2 = \pi rl$

$\frac{A - 2\pi r^2}{\pi r} = \frac{\pi rl}{\pi r}$

$\therefore l = \frac{A - 2\pi r^2}{\pi r}$

(c) $s = ut + \frac{1}{2}at^2$

$s - \frac{1}{2}at^2 = ut + \frac{1}{2}at^2 - \frac{1}{2}at^2$

$s - \frac{1}{2}at^2 = ut$

$\frac{s - \frac{1}{2}at^2}{t} = \frac{ut}{t}$

$\therefore u = \frac{s}{t} - \frac{1}{2}at$

(d) $S = \frac{n}{2}[2a + (n-1)d]$

$\frac{2}{n} \times S = \frac{2}{n} \times \frac{n}{2}[2a + (n-1)d]$

$\frac{2S}{n} = 2a + (n-1)d$

$\frac{2S}{n} - 2a = 2a + (n-1)d - 2a$

$\frac{2S - 2an}{n} = (n-1)d$

$\frac{2S - 2an}{n(n-1)} = \frac{(n-1)d}{n-1}$

$\therefore d = \frac{2S - 2an}{n(n-1)}$

8. **(a)** $\frac{1}{h+1} + 2 = k$

$\frac{1}{h+1} + 2 - 2 = k - 2$

$\frac{1}{h+1} = k - 2$

$(h+1) \times \frac{1}{h+1} = (h+1)(k-2)$

$1 = (h+1)(k-2)$

$\frac{1}{k-2} = h + 1$

$\frac{1}{k-2} - 1 = h + 1 - 1$

$\therefore h = \frac{1}{k-2} - 1$

$= \frac{1-(k-2)}{k-2}$

$= \frac{1-k+2}{k-2}$

$= \frac{3-k}{k-2}$

Q#8

(b)

$$z = \frac{y(z-y)}{x}$$
$$x \times z = x \times \frac{y(z-y)}{x}$$
$$xz = y(z-y)$$
$$xz = yz - y^2$$
$$xz - yz = yz - y^2 - yz$$
$$xz - yz = -y^2$$
$$z(x-y) = -y^2$$
$$\frac{z(x-y)}{x-y} = -\frac{y^2}{x-y}$$
$$\therefore z = -\frac{y^2}{x-y}$$
$$= \frac{y^2}{y-x}$$

(c)

$$\frac{px}{q} = p + q$$
$$q \times \left(\frac{px}{q}\right) = q \times (p+q)$$
$$px = pq + q^2$$
$$px - pq = pq + q^2 - pq$$
$$px - pq = q^2$$
$$\frac{p(x-q)}{x-q} = \frac{q^2}{x-q}$$
$$\therefore p = \frac{q^2}{x-q}$$

(d)

$$\frac{1}{a} + \frac{1}{b} = 1$$
$$ab \times \left(\frac{1}{a} + \frac{1}{b}\right) = ab$$
$$b + a = ab$$
$$b + a - b = ab - b$$
$$a = ab - b$$
$$a = b(a-1)$$
$$\frac{a}{a-1} = \frac{b(a-1)}{a-1}$$
$$\therefore b = \frac{a}{a-1}$$

9. (a)

$$V = \frac{4}{3}\pi r^3$$
$$\frac{3}{4\pi} \times V = \frac{3}{4\pi} \times \frac{4}{3}\pi r^3$$
$$\frac{3V}{4\pi} = r^3$$
$$\therefore r = \sqrt[3]{\frac{3V}{4\pi}}$$

(b)

$$v^2 = u^2 + 2as$$
$$v^2 - 2as = u^2 + 2as - 2as$$
$$v^2 - 2as = u^2$$
$$\therefore u = \pm\sqrt{v^2 - 2as}$$

(c)

$$y = (x-p)^2 + q$$
$$y - q = (x-p)^2 + q - q$$
$$y - q = (x-p)^2$$
$$\pm\sqrt{y-q} = x - p$$
$$p \pm \sqrt{y-q} = p + x - p$$
$$\therefore x = p \pm \sqrt{y-q}$$

(d)

$$t = \sqrt{\frac{4z}{m-3}}$$
$$t^2 = \frac{4z}{m-3}$$
$$t^2 \times (m-3) = \frac{4z}{m-3} \times (m-3)$$
$$t^2(m-3) = 4z$$
$$\frac{t^2(m-3)}{4} = \frac{4z}{4}$$
$$\therefore z = \frac{t^2(m-3)}{4}$$

10. (i)

$$V = \pi r^2 h + \frac{2}{3}\pi r^3$$
$$V - \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{2}{3}\pi r^3 - \frac{2}{3}\pi r^3$$
$$V - \frac{2}{3}\pi r^3 = \pi r^2 h$$
$$\frac{V - \frac{2}{3}\pi r^3}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$\therefore h = \frac{V - \frac{2}{3}\pi r^3}{\pi r^2}$$
$$= \frac{V}{\pi r^2} - \frac{2}{3}r$$

(ii) When $V = 245$ and $r = 7$,

$$h = \frac{245}{\pi(7^2)} - \frac{2}{3}(7)$$
$$= -3.08 \text{ (to 3 s.f.)}$$

11. (i)

$$a = \sqrt{\frac{3b+c}{b-c}}$$
$$a^2 = \frac{3b+c}{b-c}$$
$$a^2 \times (b-c) = 3b + c$$
$$a^2b - a^2c = 3b + c$$
$$a^2b - a^2c + a^2c = 3b + c + a^2c$$
$$a^2b = 3b + c + a^2c$$
$$a^2b - 3b = 3b + c + a^2c - 3b$$
$$a^2b - 3b = c + a^2c$$
$$b(a^2 - 3) = c + a^2c$$
$$\frac{b(a^2-3)}{a^2-3} = \frac{c + a^2c}{a^2-3}$$
$$\therefore b = \frac{c + a^2c}{a^2-3}$$

(ii) When $a = 2$ and $c = 5$,

$$b = \frac{5 + 2^2(5)}{2^2 - 3}$$
$$= \frac{5 + 20}{4 - 3}$$
$$= 25$$

12. (a) When $m = 5, n = 7, x = 4$ and $y = -2$,

$$\frac{5[(7 \times 4) - (-2)^2]}{p} = 3 \times 7$$
$$\frac{5(28 - 4)}{p} = 21$$
$$\frac{5 \times 24}{p} = 21$$
$$\frac{120}{p} = 21$$
$$120 = 21p$$
$$\therefore p = 5\frac{5}{7}$$

(b) When $m = 14, p = 9, x = 2$ and $y = 3$,

$$\frac{14[n(2) - 3^2]}{9} = 3n$$
$$\frac{14(2n - 9)}{9} = 3n$$
$$14(2n - 9) = 27n$$
$$28n - 126 = 27n$$
$$n - 126 = 0$$
$$\therefore n = 126$$

(c) When $m = 5, n = 4, p = 15$ and $x = 42$,

$$\frac{5[4(42) - y^2]}{15} = 3(4)$$
$$\frac{5(168 - y^2)}{15} = 12$$
$$5(168 - y^2) = 180$$
$$168 - y^2 = 36$$
$$132 - y^2 = 0$$
$$y^2 = 132$$
$$\therefore y = \pm\sqrt{132}$$
$$= \pm 11.5 \text{ (to 3 s.f.)}$$

13. (a) When $\pi = 3.142, h = 15$ and $r = 7$,

$$A = \frac{1}{3}(3.142)(7^2)(15) + \frac{4}{3}(3.142)(7^3)$$
$$= 769.79 + 1436.94$$
$$= 2210 \text{ (to 3 s.f.)}$$

(b) When $\pi = 3.142, A = 15\,400$ and $r = 14$,

$$15\,400 = \frac{1}{3}(3.142)(14^2)h + \frac{4}{3}(3.142)(14^3)$$
$$15\,400 = 205.277h + 11\,495.531$$
$$3904.469 = 205.277h$$
$$\therefore h = 19.0 \text{ (to 3 s.f.)}$$

14. (a)

$$2 - \frac{5}{x + 2} = 1\frac{3}{5}$$
$$10(x + 2) - 25 = 8(x + 2)$$
$$10x + 20 - 25 = 8x + 16$$
$$10x - 5 = 8x + 16$$
$$10x = 8x + 21$$
$$2x = 21$$
$$\therefore x = 10\frac{1}{2}$$

(b)

$$\frac{7x - 4}{15} + \frac{x - 1}{3} = \frac{3x - 1}{5} - \frac{7 + x}{10}$$
$$2(7x - 4) + 10(x - 1) = 6(3x - 1) - 3(7 + x)$$
$$14x - 8 + 10x - 10 = 18x - 6 - 21 - 3x$$
$$24x - 18 = 15x - 27$$
$$24x = 15x - 9$$
$$9 = -9$$
$$\therefore x = -1$$

(c)

$$\frac{x + 1}{5x - 1} + \frac{1}{2(1 - 5x)} = \frac{1}{4}$$
$$4(x + 1) - 2 = 5x - 1$$
$$4x + 4 - 2 = 5x - 1$$
$$4x + 2 = 5x - 1$$
$$4x + 3 = 5x$$
$$\therefore x = 3$$

(d)

$$\frac{5}{2x - 1} - \frac{4}{4x - 2} - \frac{3}{6x - 3} = 1$$
$$\frac{5}{2x - 1} - \frac{4}{2(2x - 1)} - \frac{3}{3(2x - 1)} = 1$$
$$\frac{5}{2x - 1} - \frac{2}{2x - 1} - \frac{1}{2x - 1} = 1$$
$$\frac{5 - 2 - 1}{2x - 1} = 1$$
$$\frac{2}{2x - 1} = 1$$
$$2 = 2x - 1$$
$$3 = 2x$$
$$\therefore x = 1\frac{1}{2}$$

(e)

$$\frac{3}{2 - x} + \frac{5}{4 - 2x} - \frac{1}{x - 2} = 4$$
$$\frac{3}{2 - x} + \frac{5}{2(2 - x)} - \frac{1}{-(2 - x)} = 4$$
$$\frac{3}{2 - x} + \frac{5}{2(2 - x)} + \frac{1}{2 - x} = 4$$
$$\frac{6}{2(2 - x)} + \frac{5}{2(2 - x)} + \frac{2}{2(2 - x)} = 4$$
$$\frac{6 + 5 + 2}{2(2 - x)} = 4$$
$$\frac{13}{2(2 - x)} = 4$$
$$13 = 8(2 - x)$$
$$13 = 16 - 8x$$
$$-3 = -8x$$
$$\therefore x = \frac{3}{8}$$