Ex # 2.3

Relation b/w roots and co-efficent of a quadratic equation

In $ax^2 + bx + c = 0$

a, b and c are coefficients of x^2, x respectively.

$\therefore \quad \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ & $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

are roots of a quadratic equation

If $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

& $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Sum of the roots

$$\alpha + \beta = -\frac{b}{a}$$

Product of the roots

$$\alpha\beta = \frac{c}{a}$$

Q#1: Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2 - 5x + 3 = 0$

Solution

Let α, β are the roots of the equation $x^2 - 5x + 3 = 0$

Then

Sum of the roots $= \alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{(-5)}{1}$$

$$\boxed{\alpha + \beta = 5}$$

Product of the roots $= \alpha\beta = \frac{c}{a}$

$$\alpha\beta = \frac{3}{1}$$

$$\boxed{\alpha\beta = 3}$$

(ii) $3x^2 + 7x - 11 = 0$

Solution:

Let α, β be the roots of the equation $3x^2 + 7x - 11 = 0$

Then

Sum of the roots $= \alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{(7)}{3}$$

$$\boxed{\alpha + \beta = -\frac{7}{3}}$$

Product of the roots $= \alpha\beta = \frac{c}{a}$

$$\boxed{\alpha\beta = -\frac{11}{3}}$$

(iii) $Px^2 - qx + r = 0$

(iv) $(a+b)x^2 - ax + b = 0$

(vi) $7x^2 - 5mx + 9n = 0$

(v) $(l+m)x^2 + (m+n)x + n - l = 0$

Solution:

Let α, β be the roots of the equation $(l+m)x^2 + (m+n)x + n - l = 0$

Then

$$\text{Sum} = \alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = -\frac{(m+n)}{l+m}$$

$$\boxed{\alpha + \beta = -\frac{m+n}{l+m}}$$

$$\text{Product} = \alpha\beta = \frac{c}{a}$$

$$\boxed{\alpha\beta = \frac{n-l}{l+m}}$$

Q#2: Find the value of K, if

(i) Sum of the roots of the equation $2Kx^2 - 3x + 4K = 0$ is twice the Product of the roots

Solution:

Let α, β be the roots of the equation $2Kx^2 - 3x + 4K = 0$

Then $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{(-3)}{2K}$$

$$\boxed{\alpha + \beta = \frac{3}{2K}}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{4K}{2K}$$

$$\boxed{\alpha\beta = 2}$$

According to condition

$$\alpha + \beta = 2(\alpha\beta)$$

$$\frac{3}{2K} = 2(2)$$

$$3 = 8K$$

$$\boxed{K = \frac{3}{8}}$$

(ii) sum of the roots of the equation $x^2 + (3K-7)x + 5K = 0$ is $\frac{3}{2}$ times the product of the roots.

Solution

Let α, β be the roots of the equation $x^2 + (3K-7)x + 5K = 0$

Then $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{(3K-7)}{1}$$

$$\boxed{\alpha + \beta = -3K + 7}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{5K}{1}$$

$$\boxed{\alpha\beta = 5K}$$

According to condition

$$\alpha + \beta = \frac{3}{2}(\alpha\beta)$$

$$-3K+7 = \frac{3}{2}(5K)$$

$$2(-3K+7) = 15K$$

$$-6K+14 = 15K$$

$$14 = 15K+6K$$

$$21K = 14$$

$$K = \frac{14}{21} \quad 2 \quad 3$$

$$\boxed{K = \frac{2}{3}}$$

#3: Find k if,

(i) Sum of the squares of the roots of the equation $4Kx^2 + 3Kx - 8 = 0$ is $\frac{2}{2}$

Solution:

Let α, β be the roots of the equation $4Kx^2 + 3Kx - 8 = 0$

Then $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = \frac{3K}{4K}$$

$$\boxed{\alpha + \beta = \frac{3}{4}}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{-2^2}{4K} = -\frac{2}{K}$$

According to condition

$$\alpha^2 + \beta^2 = 2$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 2$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 2$$

$$\left(\frac{9}{16}\right) - 2\left(\frac{-2}{K}\right) = 2$$

$$\frac{4}{K} = 2 - \frac{9}{16}$$

$$\frac{4}{K} = \frac{32 - 9}{16}$$

$$\frac{4}{K} = \frac{23}{16}$$

$$\frac{K}{4} = \frac{16}{23}$$

$$\boxed{K = \frac{64}{23}}$$

Q#8(ii) Sum of the squares of the roots is 6 of the equation $x^2 - 2kx + 2k + 1 = 0$

Solution:

Let α, β be the roots of the given equation.

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = -\frac{(-2k)}{1} \qquad \alpha\beta = \frac{2k+1}{1}$$

$$\boxed{\alpha + \beta = 2k} \qquad \boxed{\alpha\beta = 2k + 1}$$

According to condition

$$\alpha^2 + \beta^2 = 6$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 6$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 6$$

$$(2k)^2 - 2(2k+1) = 6$$

$$4k^2 - 4k - 2 - 6 = 0$$

$$4k^2 - 4k - 8 = 0$$

$$4(k^2 - k - 2) = 0$$

$$k^2 - k - 2 = 0$$

$$k^2 - 2k + 1k - 2 = 0$$

$$k(k-2) + 1(k-2) = 0$$

$(K+1)(K-2) = 0$

$K+1 = 0$ or $K-2 = 0$

$\boxed{K = -1}$ or $\boxed{K = 2}$

Q#4: Find p if

(i) the roots of the equation $x^2 - x + p^2 = 0$ differ by unity.

Solution:

Let α & $\alpha - 1$ be the roots of the equation $x^2 - x + p^2 = 0$

α $\quad$ β

Sum $= \alpha + \alpha - 1 = -\frac{b}{a}$

$2\alpha - 1 = -\frac{(-1)}{1}$

$2\alpha - 1 = 1$

$2\alpha = 2$

$\boxed{\alpha = 1}$

Product $= \alpha(\alpha - 1) = \frac{c}{a}$

$\alpha(\alpha - 1) = p^2$

Put $\alpha = 1$

$1(1-1) = p^2$

$p^2 = 0$

$\boxed{p = 0}$

(ii) the roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.

Solution:

Let $\alpha, \alpha - 2$ be the roots of the equation $x^2 + 3x + p - 2 = 0$

$$\text{Sum} = \alpha + \alpha - 2 = -\frac{b}{a}$$

$$2\alpha - 2 = -\frac{3}{1}$$

$$2\alpha - 2 = -3$$

$$2\alpha = -3 + 2$$

$$\boxed{\alpha = -\frac{1}{2}}$$

$$\text{Product} = \alpha(\alpha - 2) = \frac{c}{a}$$

$$\alpha^2(\alpha - 2) = \frac{p-2}{1}$$

Put $\alpha = -\frac{1}{2}$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 2\right) = p - 2$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1-4}{2}\right) = p - 2$$

$$\frac{5}{4} = p - 2$$

$$P = \frac{5}{4} + 2$$

$$P = \frac{5+8}{4}$$

$$\boxed{P = \frac{13}{4}}$$

Q#5: Find m, if

(i) the roots of the equation $x^2 - 7x + 3m - 5 = 0$ satisfy the relation $3\alpha + 2\beta = 4$

Solution:

Let α, β be the roots of the equation $x^2 - 7x + 3m - 5 = 0$

$$\alpha + \beta = -\frac{b}{a}$$

$$= -\frac{(-7)}{1}$$

$$\boxed{\alpha + \beta = 7} \rightarrow (1)$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{3m-5}{1}$$

$$\boxed{\alpha\beta = 3m-5} \rightarrow (2)$$

Since $3\alpha + 2\beta = 4 \rightarrow (3)$

consider eq (1)

$$\alpha = 7-\beta \rightarrow (4)$$

Put $\alpha = 7-\beta$ in equation (3)

$$3(7-\beta) + 2\beta = 4$$

$$21 - 3\beta + 2\beta = 4$$

$$21 - \beta = 4$$

$$-\beta = 4-21$$

$$-\beta = -17$$

$$\boxed{\beta = 17}$$

Put $\beta = 17$ in eq (4)

$$\alpha = 7-17$$

$$\boxed{\alpha = -10}$$

Put $\alpha = -10$ & $\beta = 17$ in equation (2)

$$(-10)(17) = 3m - 5$$

$$-170 = 3m - 5$$

$$-170 + 5 = 3m$$

$$-165 = 3m$$

$$m = \frac{-165}{3}$$

$$\boxed{m = -55}$$

(ii) the roots of the equation $x^2 + 7x + 3m - 5 = 0$ satisfy the relation $3\alpha - 2\beta = 4$

Solution:

Let α, β be the roots of the equation $x^2 + 7x + 3m - 5 = 0$.

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = -\frac{(7)}{1}$$

$$\boxed{\alpha + \beta = -7} \rightarrow (1)$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{3m-5}{1}$$

$$\boxed{\alpha\beta = 3m-5} \rightarrow (2)$$

Since $3\alpha - 2\beta = 4$ (Given) $\rightarrow (3)$

consider eq (1)

$$\alpha + \beta = -7$$

$$\alpha = -7 - \beta \rightarrow (4)$$

Put $\alpha = -7-\beta$ in eq (3)

$$3(-7-\beta) - 2\beta = 4$$

$$-21 - 3\beta - 2\beta = 4$$

$$-5\beta = 4 + 21$$

$$\beta = \frac{25}{-5}$$

$$\boxed{\beta = -5}$$

Put $\beta = -5$ in eq (1)

$$\alpha = -7 - (-5)$$

$$\alpha = -7 + 5$$

$$\boxed{\alpha = -2}$$

Put $\alpha = -2$ & $\beta = -5$ in eq (2)

$$(-5)(-2) = 3m - 5$$

$$10 = 3m - 5$$

$$10 + 5 = 3m$$

$$3m = 15$$

$$m = \frac{15}{3}$$

$$\boxed{m = 5}$$

(iii) the roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation $7\alpha - 3\beta = 18$

#6. Find m, if sum and product of roots of the following equations is equal to given number λ.

i) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Solution:

Let α, β be the roots of the equation $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

$$\alpha + \beta = \frac{-b}{a}$$

$$\boxed{\alpha + \beta = -\frac{(7m-5)}{2m+3}}$$

$$\alpha\beta = \frac{c}{a}$$

$$\boxed{\alpha\beta = \frac{3m-10}{2m+3}}$$

According to condition

$$\alpha + \beta = \alpha\beta$$

$$-\frac{(7m-5)}{2m+3} = \frac{3m-10}{2m+3}$$

$$-7m+5 = 3m-10$$

$$-7m-3m = -5-10$$

$$-10m = -15$$

$10m = 15$

$m = \frac{15}{10}$

$m = \frac{3}{2}$

$\boxed{m = \frac{3}{2}}$

(ii) $4x^2 - (3+5m)x - (9m-17) = 0$

Solution

Let α, β be the roots of the equation $4x^2 - (3+5m)x - (9m-17) = 0$

$$\alpha + \beta = -\frac{b}{a}$$

$$= -\frac{(3+5m)}{4}$$

$$\alpha\beta = \frac{c}{a}$$

$$= -\frac{(9m-17)}{4}$$

According to condition

$$\alpha + \beta = \alpha\beta$$

$$-\frac{(3+5m)}{4} = -\frac{(9m-17)}{4}$$

$3 + 5m = 9m - 17$

$5m - 9m = -3 - 17$

$-4m = -20$

$m = \frac{-20}{-4}^{5}$

$\boxed{m = 5}$