Ex #2.6

Q#1: Use synthetic division to find the quotient and the remainder when

(i) $(x^2+7x-1) \div (x+1)$

Solution

$x+1=0$

$x=-1$

	x^2	x	c
-1	1	7	-1
	↓	-1	-6
	1	6	$-7 \rightarrow R$

Quotient $= Q(x) = 1x+6$

Remainder $= R = -7$.

(ii) $4x^3-5x+15 \div x+3$

Solution:

-3	4	0	-5	15
	↓	-12	36	-93
	4	-12	31	$78 \rightarrow R$

$Q(x) = 4x^2-12x+31$

$R = 78$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

Solution:

2	1	1	-3	2
	↓	2	6	6
	1	3	3	8 → R

$Q(x) = 1x^2 + 3x + 3$

$R = 3$

Q#2: Find the value of h using synthetic division, if

(i) 3 is zero of the polynomial $2x^3 - 3hx^2 + 9$

3	2	-3h	0	9
	↓	6	18-9h	54-27h
	2	6-3h	18-9h	63-27h → R

Since 3 is zero of the polynomial therefore remainder is zero that

$63 - 27h = 0$

$63 = 27h$

$h = 63/27$

$h = 3$

$$\boxed{h = \frac{7}{3}}$$

(ii) 1 is the zero of the Polynomial $x^3 - 2hx^2 + 11$

Solution

1	1	$-2h$	0	11
	↓	1	$-2h+1$	$-2h+1$
	1	$-2h+1$	$-2h+1$	$-2h+12 \rightarrow R$

As 1 is zero of the polynomial therefore remainder is zero so

$$-2h + 12 = 0$$
$$-2h = -12$$
$$2h = 12$$
$$\boxed{h = 6}$$

(iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$

Solution

-1	2	0	$5h$	-23
	↓	-2	2	$-5h-2$
	2	-2	$5h+2$	$-5h-25 \rightarrow R$

1 is zero so remaind is zero of the polynomial

$-5h-25=0$

$-5h=25$

$\boxed{h=-5}$

#3: Use synthetic division to find the values of l & m if

$(x+3)(x-2)$ are the factors of the polynomial $x^3+4x^2+2lx+m$

-3	1	4	2l	m
	↓	-3	-3	-6l+9
	1	1	2l-3	-6l+m+9 → l
2	1	1	2l-3	
	↓	2	6	
	1	3	2l+3 → R	

1+3 & x-2 are the factors of the polynomial so remainder is zero.

$$-6l + m + 9 = 0 \quad \rightarrow (1)$$

$$2l + 3 = 0 \quad \rightarrow (2)$$

consider eq (2)

$$2l = -3$$

$$\boxed{l = -\frac{3}{2}}$$

Put $l = -\frac{3}{2}$ in eq (1)

$$-6\left(\frac{-3}{2}\right) + m + 9 = 0$$

$$+9 + m + 9 = 0$$

$$\boxed{m = -18}$$

(ii) $(x-1)$ & $(x+1)$ are the factors of the polynomial $x^3 - 3\ell x^2 + 2mx + 6$

1	1	-3ℓ	$2m$	6
	↓	~~-3ℓ~~ $+1$	$-3\ell+1$	$-3\ell+2m+1$
-1	1	$-3\ell+1$	$-3\ell+2m+1$	$-3\ell+2m+7 \to R$
	↓	-1	3ℓ	
	1	-3ℓ	$2m+1 \to R$	

As 1 & -1 are the factors of the polynomial so remainder is zero

$$2m+1 = 0 \quad \to (1)$$
$$-3\ell+2m+7 = 0 \quad \to (2)$$

consider eq (1)

$$2m+1 = 0$$
$$\boxed{m = -\frac{1}{2}}$$

Put $m = -\frac{1}{2}$ in eq (2)

$$-3\ell + 2\left(-\frac{1}{2}\right) + 7 = 0$$
$$-3\ell + 6 = 0$$
$$\boxed{\ell = 2}$$

#4: Solve by using synthetic division, if

i) 2 is the root of the equation $x^3 - 28x + 48 = 0$

Solution:

2	1	0	-28	48
	↓	2	4	-48
	1	2	-24	0

The depressed equation is

$$1x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x-4)(x+6) = 0$$

$$x = 4 \quad \& \quad x = -6$$

Thus 2, 4 & -6 are the roots of the given equation.

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

Solution

3	2	-3	-11	6
	↓	6	9	-6
	2	3	-2	0

The depressed equation is

$$2x^2 + 3x - 2 = 0$$

$$2x^2 + 4x - x - 2 = 0$$

$$2x(x+2) - 1(x+2) = 0$$

$$(2x-1)(x+2) = 0$$

$$\boxed{x = \frac{1}{2}} \quad \& \quad \boxed{x = -2}$$

(iii) -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

solution

-1	4	-1	-11	-6
	↓	-4	5	6
	4	-5	-6	0

The depressed equation is

$$4x^2 - 5x - 6 = 0$$

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x-2) + 3(x-2) = 0$$

$$(4x+3)(x-2) = 0$$

$$\boxed{x = -\frac{3}{4}} \text{ & } \boxed{x = 2}$$

#5: Solve by using synthetic division ; if

i) 1 & 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

Solution

1	1	0	-10	0	9
	↓	1	1	-9	-9
	1	1	-9	-9	0
3	↓	3	12	9	
	1	4	3	0	

The depressed equation is

$$1x^2 + 4x + 3 = 0$$

$$x^2 + 3x + x + 3 = 0$$

$$x(x+3) + 1(x+3) = 0$$

$$\boxed{x = -1} \quad \& \quad \boxed{x = -3}$$

(iii) 3 & -4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Solution

3	1	2	-13	-14	24
	↓	3	15	6	-24
-4	1	5	2	-8	0
	↓	-4	-4	8	
	1	1	-2	0	

The depressed equation is

$$x^2 + 1x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x-1)(x+2) = 0$$

$$\boxed{x = 1} \quad \& \quad \boxed{x = -2}$$