Formation of quadratic equation

If α, β be the roots of the equation $ax^2 + bx + c = 0$

Let $x = \alpha$ & $x = \beta$

$x - \alpha = 0$ $\quad$ $x - \beta = 0$

$(x - \alpha)(x - \beta) = 0$

$x^2 - \beta x - \alpha x + \alpha\beta = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - Sx + P = 0$

Ex #2.5

Q#1 Write the quadratic equation having following roots.

(a) 1, 5

Solution

Since 1 & 5 are the roots of the required quadratic equation therefore

$S = 1 + 5$ | $P = (1)(5)$

$\boxed{S = 6}$ | $P = 5$

As $x^2 - Sx + P = 0$

$x^2 - 6x + 5 = 0$ is required quadratic equation.

(b) 4, 9

(c) -2, 3

(d) 0, -3

(e) 2, -6

(f) -1, -7

(g) $1+i$, $1-i$

Solution

Since $1+i$ & $1-i$ are the roots of the required quadratic equation therefore

$S = 1+i+1-i$

$\boxed{S = 2}$

$P = (1+i)(1-i)$

$P = 1-i^2$

$P = 1-(-1) \quad \because i^2 = -1$

$\boxed{P = 2}$

As $x^2 - Sx + P = 0$

$x^2 - 2x + 2 = 0$ is required quadratic equation.

(h) $3+\sqrt{2}$, $3-\sqrt{2}$

Solution:

Since $3+\sqrt{2}$ & $3-\sqrt{2}$ are the roots of the quadratic equation therefore

$S = 3+\sqrt{2}+3-\sqrt{2}$

$\boxed{S = 6}$

$P = (3+\sqrt{2})(3-\sqrt{2})$

$P = 9-2$

$\boxed{P = 7}$

As $x^2 - Sx + P = 0$

$x^2 - 6x + 7 = 0$ is required quadratic equation.

Q#2: If α, β are the roots of the equation $x^2 - 3x + 6 = 0$

Form equation whose roots are

(a) $2\alpha+1, 2\beta+1$

Solution:

As α, β be the roots of the equation $x^2 - 3x + 6 = 0$

So $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta = -\frac{(-3)}{1}$$

$$\boxed{\alpha + \beta = 3}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{6}{1}$$

$$\boxed{\alpha\beta = 6}$$

$2\alpha+1, 2\beta+1$

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$S = 2\alpha + 1 + 2\beta + 1$

$S = 2\alpha + 2\beta + 2$

$= 2(\alpha + \beta) + 1)$

Put $\alpha + \beta = 3$

$S = 2[(3) + 1]$

$\boxed{S = 8}$

$P = (2\alpha + 1)(2\beta + 1)$

$= 4\alpha\beta + 2\alpha + 2\beta + 1$

$= 4\alpha\beta + 2(\alpha + \beta) + 1$

Put $\alpha + \beta = 3$ & $\alpha\beta = 6$

$P = 4(6) + 2(3) + 1$

$= 24 + 6 + 1$

$P = 31$

As $x^2 - Sx + P = 0$

$x^2 - 8x + 31 = 0$ is required quadratic equation.

(b) α^2, β^2

$S = \alpha^2 + \beta^2$

$= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta$

Put $\alpha + \beta = 3$ & $\alpha\beta = 6$

$S = (3)^2 - 2(6)$

$= 9 - 6$

$\boxed{S = -3}$

$P = \alpha^2\beta^2$

$P = (\alpha\beta)^2 = (6)^2 = 36$

As $x^2 - Sx + P = 0$

$x^2 + 3x + 36 = 0$ is required quardratic equation

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

$S = \frac{1}{\alpha} + \frac{1}{\beta}$

$$S = \frac{\alpha+\beta}{\alpha\beta}$$

Put $\alpha+\beta = 3$ & $\alpha\beta = 6$

$$S = \frac{3}{6}$$

$$\boxed{S = \frac{1}{2}}$$

$$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$$

$$P = \left(\frac{1}{\alpha\beta}\right)$$

Put $\alpha\beta = 6$

$$\boxed{P = \frac{1}{6}}$$

As $x^2 - Sx + P = 0$

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

$6x^2 - 3x + 1 = 0$ is required quadratic equation.

(d) $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$

$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

Put $\alpha+\beta = 3$ & $\alpha\beta = 6$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$= \frac{9-12}{6}$$

$$S = \frac{-3}{6}$$

$$\boxed{S = -\frac{1}{2}}$$

$$P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$$

$$\boxed{P = 1}$$

As $x^2 - Sx + P = 0$

$$x^2 + \frac{1}{2}x + 1 = 0$$

$$2x^2 + x + 2 = 0$$ is required quadratic equation.

(e) $\alpha+\beta, \quad \frac{1}{\alpha}+\frac{1}{\beta}$

$$\text{Sum} = \alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}$$

$$= \alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$$

Put $\alpha+\beta=3$ & $\alpha\beta=6$

$$S = (3)+\frac{3}{6}1$$

$$= 3+\frac{1}{2}$$

$$= \frac{6+1}{2}$$

$$\boxed{S = \frac{7}{2}}$$

$$P = (\alpha+\beta)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

$$= (\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

Put $\alpha+\beta=3$; $\alpha\beta=6$

$$P=\frac{(3)^2}{6}$$

$$P=\frac{3}{2}$$

As $x^2-Sx+P=0$

$$x^2-\frac{1}{2}x+\frac{3}{2}=0$$

$$\boxed{2x^2-7x+3=0}$$

Q#3: If α,β are the roots of the equation $x^2+px+q=0$ Form equation whose roots are

(a) α^2,β^2 (b) $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$

a) If α,β be the roots of the equation $x^2+px+q=0$

$$S=\alpha+\beta=-\frac{b}{a}$$

$$\boxed{\alpha+\beta=-p}$$

$$P = \frac{c}{a}$$

$$\boxed{\alpha\beta = q}$$

(a) $\alpha^2 + \beta^2$

Solution

$$S = \alpha^2 + \beta^2$$

$$= \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

Put $\alpha + \beta = -p$ & $\alpha\beta = q$

$$S = (-p)^2 - 2(q)$$

$$\boxed{S = p^2 - 2q}$$

$$P = \alpha^2\beta^2$$

$$P = (\alpha\beta)^2$$

$$= (q)^2$$

As $x^2 - Sx + P = 0$

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

Put $\alpha + \beta = p$ & $\alpha\beta = q$

$$S = \frac{p^2 - 2q}{q}$$

$$P = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$$

$$\boxed{P = 1}$$

As $x^2 - Sx + P = 0$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

$$qx^2 - (p^2 - 2q)x + q = 0$$